Se

$$x + \frac{1}{x} = 1$$

quanto vale

$$x^k + \frac{1}{x^k} = ?$$

SOLUZIONE:

$$x = \frac{1 \pm i\sqrt{3}}{2} = e^{\pm i\frac{\pi}{3}}$$

$$\frac{1}{x} = e^{\mp i\frac{\pi}{3}}$$

$$x^{\pm k} = e^{\pm i\frac{k\pi}{3}}$$

$$x^k + \frac{1}{x^k} = e^{\pm i\frac{k\pi}{3}} + e^{\mp i\frac{k\pi}{3}} = 2\cos\frac{k\pi}{3}$$

da qui

$x^2 + \frac{1}{x^2} = 2\cos\frac{2\pi}{3} = -1$ $\quad x^3 + \frac{1}{x^3} = 2\cos\pi = -2$ $\quad x^4 + \frac{1}{x^4} = 2\cos\frac{4\pi}{3} = -1$

$x^5 + \frac{1}{x^5} = 2\cos\frac{5\pi}{3} = 1$ $\quad x^6 + \frac{1}{x^6} = 2\cos 2\pi = 2$ $\quad x^7 + \frac{1}{x^7} = 2\cos\frac{7\pi}{3} = 1$

$$\cos\frac{k\pi}{3} = \cos\left(\frac{k\pi}{3} + 2\pi\right) = \cos\left(\frac{k\pi + 6\pi}{3}\right) = \cos\frac{(k+6)\pi}{3}$$

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$$x^k + \frac{1}{x^k} = x^{k+6} + \frac{1}{x^{k+6}}$$

se

$$x^2 + \frac{1}{x^2} = 1$$

quanto vale

$$x^k + \frac{1}{x^k} = ?$$

SOLUZIONE:

$$x = e^{i\omega}$$

$$x^2 = e^{2i\omega}$$

$$e^{2i\omega} + e^{-2i\omega} = 1$$

$$2\cos 2\omega = \frac{1}{2}$$

$$2\omega = \frac{\pi}{3}$$

$$\omega = \frac{\pi}{6}$$

$$x^k = e^{ik\frac{\pi}{6}}$$

$$x^k + \frac{1}{x^k} = e^{\pm i\frac{k\pi}{6}} + e^{\mp i\frac{k\pi}{6}} = 2\cos\frac{k\pi}{6}$$

da qui

$$x + \frac{1}{x} = 2\cos\frac{\pi}{6} = \sqrt{3}$$

$x^2 + \frac{1}{x^2} = 2\cos\frac{2\pi}{6} = 1$ $\quad$ $x^3 + \frac{1}{x^3} = 2\cos\frac{\pi}{2} = 0$ $\quad$ $x^4 + \frac{1}{x^4} = 2\cos\frac{2\pi}{3} = -1$

$x^5 + \frac{1}{x^5} = 2\cos\frac{5\pi}{6} = -\sqrt{3}$ $\quad$ $x^6 + \frac{1}{x^6} = 2\cos\pi = 0$ $\quad$ $x^7 + \frac{1}{x^7} = 2\cos\frac{7\pi}{6} = -\sqrt{3}$

$x^8 + \frac{1}{x^8} = 2\cos\frac{4\pi}{3} = -1$ $\quad$ $x^9 + \frac{1}{x^9} = 2\cos\frac{3\pi}{2} = 0$ $\quad$ $x^{10} + \frac{1}{x^{10}} = 2\cos\frac{5\pi}{3} = 1$

$x^{11} + \frac{1}{x^{11}} = 2\cos\frac{11\pi}{6} = \sqrt{3}$ $\quad x^{12} + \frac{1}{x^{12}} = 2\cos 2\pi = 2$ $\quad x^{13} + \frac{1}{x^{13}} = 2\cos\frac{\pi}{6} = \sqrt{3}$

$$\cos\frac{k\pi}{6} = \cos\left(\frac{k\pi}{6} + 2\pi\right) = \cos\left(\frac{k\pi + 12\pi}{6}\right) = \cos\frac{(k+12)\pi}{3}$$

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$$x^k + \frac{1}{x^k} = x^{k+12} + \frac{1}{x^{k+12}}$$

Infine. Infine?

se

$$x^q + \frac{1}{x^q} = 1$$

ABBIAMO:

$$x^q = e^{qi\omega}$$

$$e^{qi\omega} + e^{-qi\omega} = 1$$

$$\cos q\omega = \frac{1}{2}$$

$$\omega = \frac{\pi}{3q}$$

$$x^k = e^{ik\frac{\pi}{3q}}$$

$$x^k + \frac{1}{x^k} = 2\cos\frac{k\pi}{3q}$$